Deriving Derivation

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Oct 31, 2022
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"It would be better for the true physics
if there were no mathematicians on earth."
— Daniel Bernoulli

Introduction

Mathematics is a graveyard of modern physics. Natural science, motion, forces, interactions just became symbols on the paper. It is not uncommon explanation for physical effects in the textbooks – "phenomenon taking place because of mathematical theorem". Just try to imagine – universe behave in certain way because somebody wrote few symbols...

Modern physics with modern mathematics are definitely not for mortals.

But there is different aspect of mathematical science. It looks like some mathematical methods and principals which were invented hundreds years ago are quite wrong when applied to physics. Good examples are Maxwell's equations and Schrodinger equation. The solution of such equations looks good and irrefutable, but the problem is that many other solutions exist and such solutions are not provided by the math.

Vector calculus is suspicious branch of mathematics in relation to physics. Think about the following – what is the result when you add two vectors of equal length but opposite direction? The answer from mathematics is certain, result will be zero. Different situation in physics, two opposite forces not necessarily produce zero result, but could be an origin of rotation. Everything depend on the forces' points of application. It appeared that in the world of physics vectors have the beginning and the end, while in mathematics there is no beginning, all vectors have common origin, which located at zero coordinate.

The topic of current article is just one formula on the vectors' derivative.

The Formula

The formula on derivation is quite simple and looks obvious. It described in many textbooks, including [1].

Consider the identity:

$$\vec{\boldsymbol{r}} \equiv r\,\vec{\boldsymbol{r}}_1 \tag{1}$$

then

$$d\,\vec{\boldsymbol{r}} = r d\,\vec{\boldsymbol{r}}_1 + \vec{\boldsymbol{r}}_1 dr \tag{2}$$

What could possibly be wrong with that?

The expression (2) among other things used in calculation of momentum change in the system with variable mass (rocket equation).

$$d\,\vec{\boldsymbol{p}} = m d\,\vec{\boldsymbol{v}} + \vec{\boldsymbol{v}}\,dm \tag{3}$$

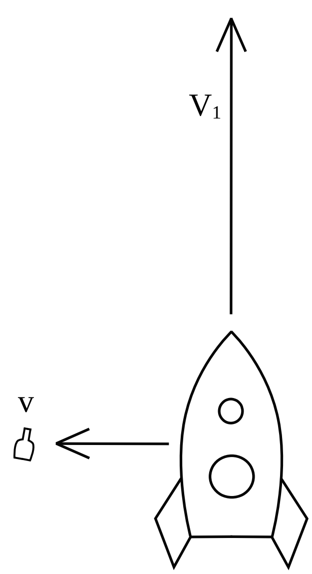
Expression (3) could also be calculated in different way:

$$d\,\vec{\boldsymbol{p}} = d(m\vec{\boldsymbol{v}}) = (m+dm)(\vec{\boldsymbol{v}}+d\,\vec{\boldsymbol{v}}) - m\,\vec{\boldsymbol{v}} = m d\,\vec{\boldsymbol{v}} + \vec{\boldsymbol{v}}\,dm + dm d\,\vec{\boldsymbol{v}} \tag{4}$$

When differentials are very small, the latter term could be omitted. Nobody never ever questioned the validity of the formula (3).

Experiment

Good point using the change in momentum for validation of the formula is the fact that we have additional relation between members. That is the momentum conservation law.



Rocket having a mass of *M*, moving through space with velocity *V*. Some small mass *m* was ejected perpendicular to the line of motion. Our goal is to calculate left and right part of the equation (3).

Momentum

Writing down momentum conservation:

$$\vec{\boldsymbol{P}}_1 = m\vec{\mathbf{v}} + \vec{\boldsymbol{P}}_2 \tag{5}$$

And

$$d\vec{\boldsymbol{P}} = \vec{\boldsymbol{P}}_2 - \vec{\boldsymbol{P}}_1 = -m\vec{\mathbf{v}} \tag{6}$$

Differentials

Good example on differentials is is calculating the circumference of the circle approximating it with polygon. The more sides polygon has, the closer its perimeter is to the circle circumference.

This is very important property of differentials. The smaller the differential is, the closer approximation we have.

Mathematics

Now we are going to calculate right part of equation (3).

The results are below and you could also download excel file.

Let V_1 = 1000, M = 1000, m = 1

Here $d\vec{\boldsymbol{R}} = md\vec{\mathbf{v}} + \vec{\mathbf{v}}\,dm$

v	|dR|/|dP|	dP to dR angle [degrees]
100	1.00005	0.57
10	1.005	5.7
1	1.415	45
0.1	10.06	84
0.01	100.1	89
0.001	1001	89.9

As you could see, $d\boldsymbol{P}$ and $d\boldsymbol{R}$ values becoming further apart with reducing the differentials.

When the changes in momentum become smaller, the difference between left and right parts of equation under question (3) became bigger. Even angle between left and right parts became bigger.

Conclusion

Formula

$$d\vec{\boldsymbol{p}} = md\vec{\mathbf{v}} + \vec{\mathbf{v}}dm \quad (3)$$

Is considered to be always true in mathematics.

In physics, when additional conditions like momentum conservation applied, the formula (3) does not hold and could not be used for calculations.

References

[1] – Joseph George Coffin (1911). Vector analysis: an introduction to vector-methods and their various applications to physics and mathematics (2nd ed.). New York. John Wiley & Sons Inc.